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New Zealand

A Mathematical Optimisation Model of a New Zealand Dairy Farm: The Integrated Dairy Enterprise (IDEA) Framework

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Working Paper in Economics 01/12

April 2012

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Abstract

Optimisation models are a key tool for the analysis of emerging policies, price sets, and technologies within grazing systems. A detailed nonlinear optimisation model of a New Zealand dairy farming system is described. The framework is notable for its rich portrayal of pasture and cow biology that add substantial descriptive power to standard approaches. Key processes incorporated in the model include: (1) pasture growth and digestibility that differ with residual pasture mass and rotation length, (2) pasture utilisation that varies by stocking rate, and (3) different levels of intake regulation. Model output is shown to closely match data from a more detailed simulation model (deviations between 0 and 5 per cent) and survey data (deviations between 1 and 11 per cent), providing confidence in its predictive capacity. Use of the model is demonstrated in an empirical application investigating the relative profitability of production systems involving different amounts of imported feed under price variation. The case study indicates superior profitability associated with the use of a moderate level of imported supplement, with Operating Profit ($NZ ha^{-1}) of 934, 926, 1186, 1314, and 1093 when imported feed makes up 0, 5, 10, 20 and 30 per cent of the diet, respectively. Stocking rate and milk production per cow increase by 35 and 29 per cent, respectively, as the proportion of imported feed increases from 0 to 30 per cent of the diet. Pasture utilisation increases with stocking rate. Accordingly, pasture eaten and nitrogen fertiliser application increase by 20 and 213 per cent, respectively, as the proportion of imported feed increases from 0 to 30 per cent of the diet.

JEL Classification

Q12, C61

Keywords

dairy production
mathematical optimisation
whole-farm model

Acknowledgements

The authors would like to acknowledge the assistance of Pierre Beukes, Jeremy Bryant, Chris Glassey, Pablo Gregorini, Ross Kingwell, Gil Levy, David McCall, David Pannell, and Nicolas Villalobos-Lopez.

1. Introduction

Dairy production is a major global industry, with annual production of around 700 million tonnes in 2010 (CDIC, 2011). Farming systems on which grazed pasture supplies the majority of energy to dairy cows are widely distributed, particularly in the temperate regions of Australia, New Zealand, and Western Europe. The potential of grazing systems to increase global market share is high, given the increasing costs associated with high levels of supplementation and environmental and welfare concerns associated with intensive dairy production (Dillon *et al.* 2005). Indeed, the total costs of milk production worldwide have been shown to decline linearly as the proportion of grass contained in cow rations increases (Dillon, 2006). Nevertheless, pasture-based dairy farms are complex systems in which producers must consider multiple interactions between pasture growth and decay, supplement use, individual animal intake and efficiency, and herd size and structure.

Optimisation models are a valuable tool in farm modelling given their ability to consider together the many interdependent elements of an agricultural system (McCall *et al.* 1999; Kingwell and Fuchsbichler 2011). Such frameworks have multiple uses, but primary applications are the assessment of agricultural innovations, evaluation of alternative management practices, research prioritisation, and policy analysis (Pannell 1996). The continued relevance of optimisation models in agricultural modelling is promoted by the flexibility of mathematical programming frameworks (Howitt 1995), the complexity of modern farming systems (Kingwell 2011), and the ongoing development of efficient algorithms and computer processing capacity (Doole 2010). Optimisation frameworks are particularly useful in the analysis of grazing systems, as these models allow for the inclusion of considerable detail, particularly regarding pasture dynamics and livestock efficiency, as well as allowing optimal solutions to be found efficiently (Doole and Pannell 2008).

A number of optimisation models have been developed for pasture-based dairy systems throughout the world. Olney and Kirk (1989) present a small linear programming (LP) model of a Western Australian dairy farm in which pasture growth and quality is fixed in each period. Pasture can either be grazed or deferred to the next period. Berentsen and Giesen (1995) describe a LP model of a Netherlands dairy system in which pasture growth is defined in terms of an annual total and its quality is fixed. However, this model was extended to incorporate three growing periods for pasture in Berentsen *et al.* (2000). McCall *et al.* (1999) present a comprehensive LP model of a dairy farming system in which the length of grazing rotations is optimised. However, cow intakes and pasture residuals, digestibility, and growth are fixed in each period to maintain tractability. Cabrera *et al.* (2005) employ LP in a dynamic simulation model to identify optimal strategies in a dairy system in Florida, U.S.A. Though detailed in many aspects, this application does not represent pasture grazing strategies with much complexity, given the high level of confinement and use of intensive supplementation in these systems. Neal *et al.* (2007) used a detailed LP model to identify the most profitable mix of 36 alternative forage combinations on a farm in New South Wales, Australia. However, the focus on the evaluation of alternative forages meant that forage residuals, digestibility, and growth were fixed in each period. Doole (2010) extended the model of McCall *et al.* (1999) to incorporate a link between production and nitrate emissions from multiple farms. However, this work retained fixed pasture residuals, digestibility, and growth to maintain tractability and reduce data requirements.

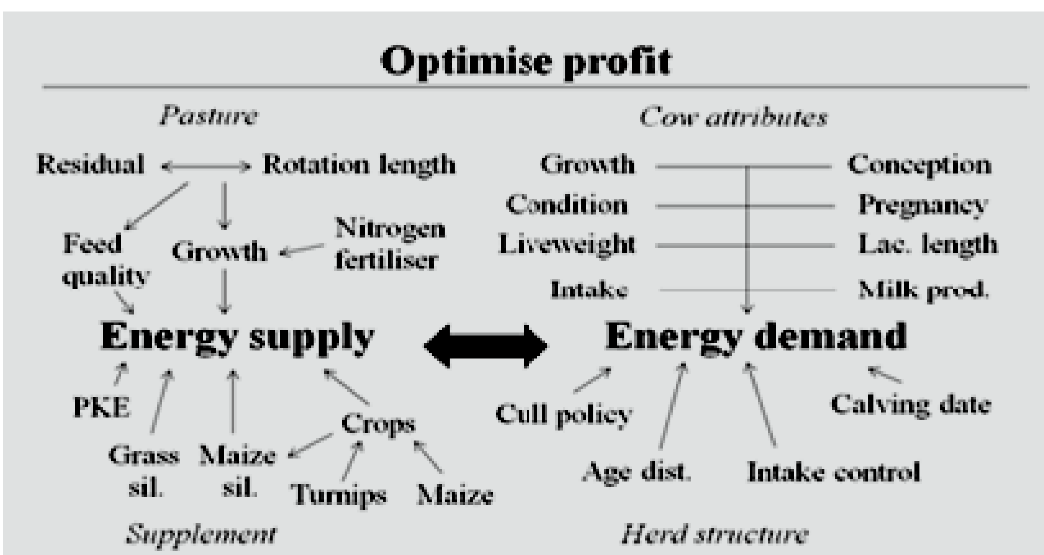
The objective of this paper is to present a detailed nonlinear programming (NLP) model of a New Zealand dairy farm that incorporates a detailed description of important processes within pasture-based dairy systems that are not considered in previous frameworks. This model - the Integrated Dairy Enterprise Analysis (IDEA) framework - is the first optimisation model of a grazing system to consider, both independently and together: (1) post-grazing residual mass as a decision variable of the producer, (2) pasture growth and digestibility that differ with residual pasture mass and rotation length, (3) pasture utilisation that varies by stocking rate, (4) inclusion of nonlinear functions describing substitution rates, and (5) different levels of intake regulation. These extensions add substantial descriptive power and eliminate critical gaps evident in linear models of grazing systems. This descriptive power of the model is demonstrated in Section 4, where model output is shown to closely match data from surveys and a complex simulation model.

The paper is organised as follows. Section 2 provides a technical description of the model. Section 3 describes the data and assumptions used in the model. Section 4 presents model output for a number of alternative scenarios. Section 5 concludes.

2. Model Description

This section describes a nonlinear optimisation model of a NZ dairy farming system. It provides detailed insight into the most-profitable balance of feed demand and feed supply across a typical milking season. All variables are listed as upper-case letters, while coefficients and parameters are denoted as lower-case letters. The structure of the model is shown in Figure 1.

Figure 1. Stylised structure of the IDEA model



2.1 Land use module

The model involves a single management year defined from 1 July to 30 June. The year is divided into 26 fortnights to provide insight into the temporal allocation of feed. Time periods within the single year are defined in a cyclical fashion, such that the first period follows the last in a continuous, repeatable fashion. This allows for the analysis of decisions that span the last and first time periods, such as the area allocated to saved pasture.

NZ dairy farmers typically employ a rotational-grazing system in which pasture is grazed intensively at very high stocking intensity and then rested. The time required to graze the entire farm with an optimum grazing strategy (rotation length) depends on the ability of a producer to manage pasture levels to maintain growth and quality, while ensuring adequate supplies of pasture are available in the future (Macdonald and Penno 1998). IDEA incorporates these decisions through allowing the optimisation of both rotation length and post-grazing residual mass. Focus on post-grazing residual mass is justified because this greatly impacts future pasture growth and quality. For example, grazing to a low residual (e.g. 0.8 t DM ha^{-1}) can severely harm regrowth, while grazing to a high residual (for example, 2.6 t DM ha^{-1}) will have a major negative impact on pasture quality in future grazing events.

The length of a fortnight is $\delta = 14$ days. Time index $i = [1, 2, ..., 26]$ denotes the fortnight in which an area of pasture was previously grazed or harvested for silage. In comparison, time index *t*, where $t = [1, 2, ..., 26]$, denotes the fortnight in which an area of pasture is currently grazed, harvested for silage, or rested for future use. An additional index $u = [1, 2, ..., 26]$ is used where a (future) activity occurs in a period past *t*.

Two residual indices are defined:

- r_i : The pasture mass (t DM ha^{-1}) that exists after the paddock was grazed or cut for silage in period *i*.
- r_t : The pasture mass (t DM ha^{-1}) that exists after the paddock was grazed or cut for silage in period *t*.

The set of potential residuals is the same for both indices, with $r_i = r_t = \{0.8, 1, ..., 2.6\}$ t DM ha^{-1}. However, r_i can be—and typically is—different from r_t for any grazing strategy.

Two variables denote the area utilised over fortnight *t* for grazing or silage production:

- A^G_{i,t,r_i,r_t} : The area (ha) of pasture grazed at time *t* to a post-grazing residual of r_t that was last grazed or ensiled in period *i* to a residual of r_i.. This decision variable drives pasture eaten from the grazing rotation.
- A^S_{i,t,r_i,r_t} : The area (ha) of pasture ensiled at time *t* to a post-ensilement residual of r_t that was grazed or ensiled in period *i* to a residual of r_i.

Consistent residual lengths require:

$$\sum_i \sum_{r_i} \left[A^G_{i,t,r_i,r_t} + A^S_{i,t,r_i,r_t} \right] = \sum_u \sum_{r_u} \left[A^G_{t,u,r_t,r_u} + A^S_{t,u,r_t,r_u} \right]. \quad (1)$$

This equation defines the land use cycle, with land used in period *t* representing the future land use for period *i* (left hand side of eq. 1), but also the previous land use for period *u* (right hand side of eq. 1).

Decision variables denote the area (ha) allocated to turnips (*TU*) and maize (*MA*). The first is a forage crop, while maize is harvested for silage. All crops are regrassed after they have been utilised. A proportion of New Zealand dairy farms is also regrassed each year without crop establishment, primarily to replace degraded pasture. The area (ha) on which this is done is denoted *RG*. The total area that is regrassed each year (*AR*) after crops or degraded pastures are removed is:

$$AR = TU + MA + RG \,. \tag{2}$$

Land use allocation in any period *t* is:

$$\begin{aligned} c_{z1} = AR + \sum_i \sum_{r_i} \sum_{r_t} \left[A^G_{i,t,r_i,r_t} + A^S_{i,t,r_i,r_t} \right] + \\ \sum_i \sum_u \sum_{r_i} \sum_{r_u} \left[A^G_{i,u,r_i,r_u} + A^S_{i,u,r_i,r_u} \right]_{\forall i \neq t, t>i, u>t, u>i} + \\ \sum_i \sum_u \sum_{r_i} \sum_{r_u} \left[A^G_{i,u,r_i,r_u} + A^S_{i,u,r_i,r_u} \right]_{\forall i \neq t, i>t, u>t, i>u} + \\ \sum_i \sum_u \sum_{r_i} \sum_{r_u} \left[A^G_{i,u,r_i,r_u} + A^S_{i,u,r_i,r_u} \right]_{\forall i \neq t, t>i, t>u, i>u} \end{aligned} , \tag{3}$$

where total farm area is c_{z1}. The definition of all coefficients for this module and the next is listed in Table 1. The estimation of coefficients is discussed in Section 3.

Table 1. Standard values of coefficients in the land use and pasture modules

Symbol	Coefficient	Unit	Value	Source
c_{z1}	Farm size	ha	$c_{z1} = 125$	DairyNZ (2011) and expert opinion
c_{z2}	Proportion of farm regrassed each year	-	$c_{z2} = 0.1$	Expert opinion
c_{g1}	Maximum application of N fertiliser each year	t ha^{-1}	$c_{g1} = 0.4$	McCall *et al.* (1999)
c_{g2}	Maximum application of N fertiliser in each period	t ha^{-1}	$c_{g2} = 0.05$	McCall *et al.* (1999)
c_{g3}	Maximum application of N fertiliser in a 6 week period	t ha^{-1}	$c_{g3} = 0.1$	McCall *et al.* (1999)
c_{g4}	Factor that converts digestibility parameters into MJ ME	-	$c_{g4} = 15000$	Joyce *et al.* (1975)

The first line in eq. 3 accounts for the area removed from the grazing rotation (*AR*) and current land use. The second and third lines define land that is being rested for future use. The second line describes cases where $u > t > i$. One example is where land was last used in period 5, it is currently period 13, and it will be utilised once again in period 17 ($u = 17 > t = 13 > i = 5$). The third line describes cases where $i > u > t$. The last line describes cases where $t > i > u$. The third and fourth lines define cases where pasture is rested for a period encompassing both the last and first fortnights, consistent with the equilibrium structure of the model.

A certain proportion of the farm must be regrassed (c_{z2}) each year, in accordance with industry practice:

$$AR \geq c_{z1}c_{z2} . \tag{4}$$

Pasture can be grazed prior to crops or new pasture being sown and after new pasture has successfully established following these land uses. Consistency in rotational management (eq. 1) complicates the removal of land for different periods for cropping or regrassing. Thus, following McCall *et al.* (1999), the area used for this purpose is removed from the grazing rotation for the entire year (eq. 3). However, the pasture produced when grass is present is accounted for (see eq. 7) to reduce bias. The area of pasture available within area *AR* in period *t* is:

$$A_t^R = TU_{t=[23,7]} + MA_{t=[23,7]} + RG_{t=[23,18]} , \tag{5}$$

where $AR \geq A_t^R$ and subscripts on the right-hand side denote the periods that pasture is available on land allocated to a given crop or regressing activity.

2.2 Pasture module

The grazing rotation yields a base amount of pasture (t DM) eaten:

$$Q_t^G = \sum_i \sum_{r_i} \sum_{r_t} A_{i,t,r_i,r_t}^G q_{i,t,r_i,r_t}^G , \tag{6}$$

where q_{i,t,r_i,r_t}^G is herbage mass available for a given set of residuals (t DM ha^{-1}).

Pasture produced on the cropped or regrassed area *AR* is specified as Q_t^R (t DM). It is computed:

$$Q_t^R = A_t^R q_t^R , \tag{7}$$

where q_t^R is an exogenous parameter denoting average pasture production in period *t* (t DM ha^{-1}). This specification assumes that all growth is grazed in each period. Residual mass is not studied specifically on area *AR* (cf. eqs. 6 and 7) to maintain model tractability, while still allowing the grazing of pasture on this land, when appropriate.

The variable N_i denotes the tonnes of nitrogen (N) fertiliser applied in time *i*. The total amount of N fertiliser applied over the farm in the year is $NF = \sum_{t=1}^{26} N_t$. The condition $c_{z1}c_{g1} \geq NF$ restricts total application, where c_{g1} is the maximum application of N fertiliser per hectare per year. The constraint $c_{g2} \geq N_t \ / \ c_{z1} \ \forall t$ restricts the application rate per hectare in each time period, where c_{g2} is the maximum application of N fertiliser per hectare per period. The condition $c_{g3} \geq \sum_{\chi=t}^{t+2} N_{\chi} / c_{z1} \; \forall t$ restricts total application rate per hectare in each six-week period, where c_{g3} is the maximum application per six-week period.

The total amount of herbage mass available to grazing animals in period *t* arising from the application of N_i (Q_t^N) in period *i* is:

$$Q_t^N = \sum_{i=1}^{26} f_{i,t} N_i \ , \tag{8}$$

where $f_{i,t}$ is the pasture mass obtained in period *t* following the application of fertiliser in period *i* (t DM ha^{-1}/t N). Following McCall *et al.* (1999), additional pasture growth from N fertiliser application is represented separately from the rotational-grazing system to reduce the number of decision variables.

Total herbage mass (t DM) grazed in time *t* is:

$$Q_t^H = Q_t^G + Q_t^R + Q_t^N \ . \tag{9}$$

Eq. 9 shows there are three pasture feed pools, though Q_t^G is by far the largest given the small proportion of feed obtained from N fertiliser application and the small area cropped or regrassed annually on NZ dairy farms (around 10 per cent).

The total amount of herbage mass (t DM) ensiled in period *t* is:

$$Q_t^S = \sum_i \sum_{r_i} \sum_{r_t} A_{i,t,r_i,r_t}^S q_{i,t,r_i,r_t}^S \ , \tag{10}$$

where q_{i,t,r_i,r_t}^S is the herbage mass (t DM ha^{-1}) available. Silage production can leave residuals of $r_t = [1.4, 1.6, ..., 2.2]$ t DM ha^{-1}. Thus, $A_{i,t,r_i,r_t \neq [1.4,1.6,...,2.2]}^S = 0$.

Pasture is rarely grazed or ensiled at intervals greater than 100 days on NZ dairy farms. Thus, $q_{i,t,r_i,r_t}^{\alpha} = 0$ and $A_{i,t,r_i,r_t}^{\alpha} = 0$ for $\alpha = \{G, S\}$ if $t - i > 7 \Leftrightarrow t > i$ or $(26 - i) + t > 7 \Leftrightarrow i > t$.

The grazing rotation yields energy (EP_t^G) (MJ ME):

$$EP_t^G = \sum_i \sum_{r_i} \sum_{r_t} A_{i,t,r_i,r_t}^G q_{i,t,r_i,r_t}^G d_{i,t,r_i,r_t}^G c_{g4} \ , \tag{11}$$

where d_{i,t,r_i,r_t}^G is the digestibility of feed (defined over $[0,1]$) and c_{g4} is a factor that converts this into the energy content of pasture (MJ ME t^{-1} DM^{-1}). The biomass and digestibility parameters in eq. 11 must be estimated for multiple scenarios, but is complicated by a lack of experimental data covering a good range of time periods and residuals over several years. Thus, a detailed model of a pasture system is used for this purpose (Section 3.1).

The energy content of pasture on the area removed for cropping and regrassing (EP_t^R) is:

$$EP_t^R = Q_t^R \left[\frac{EP_t^G}{Q_t^G} \right]. \tag{12}$$

The term in square brackets defines mean energy (MJ ME t^{-1} DM^{-1}) computed for the major source of pasture for livestock—the grazing rotation—to reduce bias accruing to the use of exogenous information. The energy content of pasture obtained from N fertiliser application (EP_t^N) is similarly:

$$EP_t^N = Q_t^N \left[\frac{EP_t^G}{Q_t^G} \right]. \tag{13}$$

2.3 Supplement module

A number of supplementary feeds are also available. These are grass silage made on-farm, maize silage made on-farm or purchased, or palm kernel expeller (PKE) that is purchased. Turnip crops can also be grazed for forage. The total amount of grass silage produced on the farm in each period is Q_t^S. The amount of grass silage eaten by cows (t DM) in each period is F_t^{SG} on the grazing area and F_t^{SP} on a feedpad. The total amount of grass silage eaten is $F_t^S = F_t^{SG} + F_t^{SP}$. The amount of grass silage sold is denoted *GSS*. Supply and demand is balanced through:

$$\sum_{t=1}^{26} Q_t^S (1-c_{s1}) = \frac{\sum_{t=1}^{26} F_t^{SG}}{(1-c_{s2})} + \frac{\sum_{t=1}^{26} F_t^{SP}}{(1-c_{s3})} + GSS\ , \tag{14}$$

where c_{s1} is a constant representing the loss of grass silage at harvest and storage, c_{s2} is the proportion of grass silage lost at feeding on the grazing area, and c_{s3} is the proportion of grass silage lost at feeding on the feed pad. A list of all coefficients for this section is given in Table 2.

The tonnes of maize silage purchased (sold) is denoted *MAP* (*MAS*). The amount of maize silage eaten by cows (t DM) in each period is F_t^{MG} on the grazing area and F_t^{MP} on a feedpad. The total amount of maize silage eaten is $F_t^M = F_t^{MG} + F_t^{MP}$. Supply and demand is balanced through:

$$(1-c_{s4})c_{s5}MA + MAP = \frac{\sum_{t=1}^{26} F_t^{MG}}{(1-c_{s6})} + \frac{\sum_{t=1}^{26} F_t^{MP}}{(1-c_{s7})} + MAS\ , \tag{15}$$

where c_{s4} is the loss of maize silage at harvest and storage, c_{s5} is the total yield of the maize crop (t DM), c_{s6} is the proportion of maize silage lost at feeding on the grazing area, and c_{s7} is the proportion of maize silage lost at feeding on the feedpad.

The amount of PKE purchased is denoted *PAP*. The amount of PKE eaten by cows (t DM) in each period is F_t^{PG} on the grazing area, F_t^{PP} on a feedpad, and F_t^{PS} in a dairy-shed feeding system. The total amount of PKE eaten is $F_t^P = F_t^{PG} + F_t^{PP} + F_t^{PS}$. Its use is governed by:

$$PAP = \frac{\sum_{t=1}^{26} F_t^{PG}}{(1-c_{s8})} + \frac{\sum_{t=1}^{26} F_t^{PP}}{(1-c_{s9})} + \frac{\sum_{t=1}^{26} F_t^{PS}}{(1-c_{s10})}\ , \tag{16}$$

where c_{s8} is the proportion of concentrate lost at feeding on the grazing area, c_{s9} is the proportion of concentrate lost at feeding on the feedpad, and c_{s10} is the proportion of concentrate lost at feeding in the dairy-shed feeding system.

Table 2. Standard values of coefficients in the supplement module

Symbol	Coefficient	Unit	Value	Source
c_{s1}	Proportional loss of grass silage at harvest and storage	-	$c_{s1} = 0.15$	Hedley (2007)
c_{s2}	Proportional loss of grass silage at feeding on paddock	-	$c_{s2} = 0.25$	Hedley (2007)
c_{s3}	Proportional loss of grass silage at feeding on feedpad	-	$c_{s3} = 0.15$	Hedley (2007)
c_{s4}	Proportional loss of maize silage during harvest and storage	-	$c_{s4} = 0.15$	Hedley (2007)
c_{s5}	Total yield of maize crop	t DM ha^{-1}	$c_{s5} = 23$	Expert opinion
c_{s6}	Proportional loss of maize silage at feeding on paddock	-	$c_{s6} = 0.25$	Hedley (2007)
c_{s7}	Proportional loss of maize silage at feeding on feedpad	-	$c_{s7} = 0.15$	Hedley (2007)
c_{s8}	Proportional loss of PKE at feeding on paddock	-	$c_{s8} = 0.30$	Hedley (2007), DairyNZ (2010)
c_{s9}	Proportional loss of PKE at feeding on feedpad	-	$c_{s9} = 0.15$	Hedley (2007), DairyNZ (2010)
c_{s10}	Proportional loss of PKE at feeding in dairy shed	-	$c_{s10} = 0.05$	Hedley (2007), DairyNZ (2010)
c_{s11}	Yield of turnip crop	t DM ha^{-1}	$c_{s11} = 10.5$	Expert opinion
c_{s12}	Proportional loss of turnips at feeding	-	$c_{s12} = 0.25$	Expert opinion
c_{s13}	Energy content of grass silage	MJ ME t^{-1} DM^{-1}	$c_{s13} = 10000$	DairyNZ (2010)
c_{s14}	Energy content of maize silage	MJ ME t^{-1} DM^{-1}	$c_{s14} = 10500$	DairyNZ (2010)
c_{s15}	Energy content of PKE	MJ ME t^{-1} DM^{-1}	$c_{s15} = 11000$	DairyNZ (2010)
c_{s16}	Energy content of turnips	MJ ME t^{-1} DM^{-1}	$c_{s16} = 12500$	DairyNZ (2010)
c_{s17}	Maximum prop. of diet that can consist of maize silage	-	$c_{s17} = 0.4$	DairyNZ (2010)
c_{s18}	Maximum prop. of diet that can consist of PKE	-	$c_{s18} = 0.3$	DairyNZ (2010)
c_{s19}	Maximum prop. of diet that can consist of PKE fed in shed	-	$c_{s19} = 0.1$	Expert opinion
c_{s20}	Maximum prop. of diet that can consist of turnips	-	$c_{s20} = 0.3$	DairyNZ (2010)

The amount of turnips eaten (t DM) in period t is F_t^T. Supply and demand is balanced by:

$$c_{s11} TU = \frac{\sum_{t=15}^{19} F_t^T}{(1 - c_{s12})}, \qquad (17)$$

where c_{s11} is the total yield of the turnip crop (t DM) and c_{s12} is the proportion of the crop not utilised at feeding, The total energy content of supplementary feed and forage crops (ES_t) is defined:

$$ES_t = c_{s13}F_t^S + c_{s14}F_t^M + c_{s15}F_t^P + c_{s16}F_t^T, \quad (18)$$

where c_{s13}, c_{s14}, c_{s15}, and c_{s16} denote the energy content (MJ ME t^{-1} DM^{-1}) of grass silage, maize silage, PKE, and turnips, respectively.

Three measures of total feed quantity are used. The first feed quantity variable is total feed consumed IN_t^{FE} (t DM) that accounts for utilisation at feeding:

$$IN_t^{FE} = Q_t^G + Q_t^R + Q_t^N + F_t^S + F_t^M + F_t^P + F_t^T. \quad (19)$$

Utilisation is considered through eqs. 14–17. The second feed quantity variable is total feed offered IN_t^{FO} (t DM). In addition to feed consumed (eq. 19), it includes feed that is wasted in the feeding process:

$$IN_t^{FO} = Q_t^G + Q_t^R + Q_t^N + \frac{F_t^{SG}}{(1-c_{s2})} + \frac{F_t^{SP}}{(1-c_{s3})} + \frac{F_t^{MG}}{(1-c_{s6})} + \frac{F_t^{MP}}{(1-c_{s7})} + \frac{F_t^{PG}}{(1-c_{s8})} + \frac{F_t^{PP}}{(1-c_{s9})} + \frac{F_t^{PS}}{(1-c_{s10})} + \frac{F_t^{T}}{(1-c_{s12})}. \quad (20)$$

The third feed quantity variable is the total feed consumed in pasture equivalent DM. It is used to account for intake substitution in eq. 48. This is denoted IN_t^{PE} and accounts for substitution rates between feeds:

$$IN_t^{PE} = Q_t^G + Q_t^R + Q_t^N + SR_t^S F_t^S + SR_t^M F_t^M + SR_t^P F_t^P + SR_t^T F_t^T, \quad (21)$$

where SR_t^S, SR_t^M, SR_t^P, and SR_t^T are substitution rates for grass silage, maize silage, PKE, and turnips, respectively. Substitution rates are calculated in eq. 44.

Dairy farms in New Zealand are classified according to the proportion of imported supplement they use (DairyNZ 2010). Production systems 1, 2, 3, 4, and 5 import 0, 4–14, 10–20, 20–30, and 30–50 per cent of total feed offered, respectively. The relevant production system to which a farm belongs is defined exogenously by the user.

Different production systems are represented through:

$$PS^L \le \frac{MAP + PAP}{\sum_{t=1}^{26} IN_t^{FO}} \le PS^U, \quad (22)$$

where PS^L and PS^U are the lower and upper bound, respectively, of the proportion of total feed offered that consists of imported supplement in a given production system.

Maximum bounds are set on the consumption of some supplementary feeds. These are set according to industry recommendations reported in DairyNZ (2010). The use of maize silage in each period is constrained through:

$$c_{s17} IN_t^{FE} \ge F_t^M, \quad (23)$$

where c_{s17} is the maximum proportion of the diet that can consist of maize silage in each period. The use of PKE in each period is constrained through:

$$c_{s18} IN_t^{FE} \ge F_t^P, \quad (24)$$

where c_{s18} is the maximum proportion of the diet that can consist of PKE in each period.

Feeding in the dairy shed is constrained by the time spent by cows being milked:

$$c_{s19} IN_t^{FE} \geq F_t^{PS} , \tag{25}$$

where c_{s19} is the maximum proportion of the diet that cows can consume during milking.

The use of turnips in each period is constrained through:

$$c_{s20} IN_t^{FE} \geq F_t^{T} , \tag{26}$$

where c_{s20} is the maximum proportion of the diet that can consist of turnips in each period.

2.4 Cow module

Six indices are used to represent differences between cows. These are:

- $uf = [1, 2, ..., 7]$: These correspond to 0, 5, 10, 15, 20, 25, and 30 per cent decreases, respectively, in the annual energy intake of a fully-fed cow.
- $a = [1, 2, ..., 4]$: These represent cows that are 2, 3, 4, and 5+ years of age, respectively.
- $m = [1, 2, ..., 5]$: These represent the genetic merit of cows according to peak milk yield (kg day^{-1}). Levels denoted by *m* are 2 standard deviations (SD) below average (27.49 l day^{-1}), 1 SD below average (29.41 l day^{-1}), average (30.38 l day^{-1}), 1 SD above average (31.36 l day^{-1}), and 2 SD above average (33.27 l day^{-1}).
- $cu = [1, 2]$: These represent standard cows ($cu = 1$) or those to be sold as culls after lactation is complete ($cu = 2$).
- $cd = [1, 2, ..., 9]$: These represent calving dates of 1 July, 15 July, 29 July, 12 August, 26 August, 9 September, 23 September, 7 October, and 21 October.
- $ll = [1, 2, ..., 7]$: These represent lactation lengths of 180, 210, 250, 265, 280, 295, and 310 days, respectively.

The number of cows with each attribute combination is denoted $C_{uf,a,m,cu,cd,ll}$. The total number of cows (*TC*) is defined:

$$TC = \sum_{uf=1}^{7} \sum_{a=1}^{4} \sum_{m=1}^{5} \sum_{cu=1}^{2} \sum_{cd=1}^{9} \sum_{ll=1}^{7} C_{uf,a,m,cu,cd,ll} . \tag{27}$$

The total number of cull cows is:

$$TCU = \sum_{uf=1}^{7} \sum_{a=1}^{4} \sum_{m=1}^{5} \sum_{cd=1}^{9} \sum_{ll=1}^{7} C_{uf,a,m,cu=2,cd,ll} . \tag{28}$$

The stocking rate (*SR*) (cows ha^{-1}) is defined:

$$SR = TC / c_{z1} . \tag{29}$$

The number of cows in each age class is determined through:

$$\sum_{cu=1}^{2} C_{uf,a+1,m,cu,cd,ll} = C_{uf,a,m,cu=1,cd,ll} SV \quad \forall a = [1, 3], \tag{30}$$

where *SV* is the survival rate. The survival rate is defined:

$$SV = 1 - c_{a1} - c_{a2} - ER , \tag{31}$$

where c_{a1} and c_{a2} are the cull for disease and natural mortality rates for adult cows and *ER* is the empty rate. Eq. 31 describes that all empty cows are culled. A list of the coefficients for this module and the next 2 modules is provided in Table 3.

Table 3. Standard values of coefficients in the cow and integration modules

Symbol	Coefficient	Unit	Value	Source
c_{a1}	Cull rate for disease in adult cows	-	$c_{a1} = 0.045$	Villalobos-Lopez *et al.* (2000)
c_{a2}	Natural mortality rate of adult cows	-	$c_{a2} = 0.015$	Villalobos-Lopez *et al.* (2000)
c_{a3}	Proportion of female calves born	-	$c_{a3} = 0.5$	Villalobos-Lopez *et al.* (2000)
c_{a4}	Natural mortality rate of calves	-	$c_{a4} = 0.04$	Villalobos-Lopez *et al.* (2000)
c_{a5}	Natural mortality rate of yearlings	-	$c_{a5} = 0.03$	Villalobos-Lopez *et al.* (2000)
c_{a6}	Planned start of calving	Fortnight	$c_{a6} = [1,5]$	User defined
c_{a7}	Convert t DM to kg DM	-	$c_{a7} = 1000$	Stockdale (2000)
c_{a8}	Convert days to fortnights	-	$c_{a8} = 0.071$	Stockdale (2000)
c_{a9}	Intercept	-	$c_{a9} = -0.14$	Stockdale (2000)
c_{a10}	Slope for pasture intake	-	$c_{a10} = 0.17$	Stockdale (2000)
c_{a11}	Slope for season index	-	$c_{a11} = 0.08$	Stockdale (2000)
c_{a12}	Slope coefficient for supplement intake	-	$c_{a12} = 0.03$	Stockdale (2000)
c_{a13}	Slope for type of supplement	-	$c_{a13}^{S/M} = c_{a13}^{P} = -0.04$	Stockdale (2000)
c_{e1}	Shape parameter for intake function	-	$c_{e1} = 0.01112$	Gregorini *et al.* (2009)
c_{e2}	Parameter describing impact of days since calving on intake function	-	$c_{e2} = -0.00346$	Gregorini *et al.* (2009)
c_{f1}	Indicates use of a feed pad	-	$c_{f1} = \{0,1\}$	User defined
c_{f2}	Indicates use of a stand-off pad	-	$c_{f2} = \{0,1\}$	User defined
c_{f3}	Max. proportion of day spent on feed pad	-	$c_{f3} = 0.1$	Expert opinion
c_{f4}	Max. proportion of day spent on stand-off pad without feed pad	-	$c_{f4} = 0.5$	Expert opinion
c_{f5}	Shape parameter for relative pasture intake function	-	$c_{f5} = -0.203$	Estimated using data from Clark *et al.* (2010)
c_{f6}	Hours in the day	Hours	$c_{f6} = 24$	-

The empty rate is determined through:

$$ER = \left[\sum_{uf=1}^{7} \sum_{a=1}^{4} \sum_{m=1}^{5} \sum_{cd=1}^{9} \sum_{ll=1}^{7} \left(\frac{C_{uf,a,m,cu=1,cd,ll}}{TC - TCU} \right) er_{uf,a,m,cd,ll} \right], \tag{32}$$

where $er_{uf,a,m,cd,ll}$ is the empty rate for a given cow type and the ratio term weights the empty rate according to the prevalence of each cow type. The empty rate $er_{uf,a,m,cd,ll}$ is computed based on the number of heats exhibited during the mating period, which depends on age, changes in cow liveweight prior to mating, and the conception rate on each heat (Beukes *et al.* 2010).

The total number of female calves sold (VFS) is:

$$VFS = TC(1-ER)c_{a3}(1-c_{a4})PS^{VF}, \tag{33}$$

where c_{a3} is the proportion of females born, c_{a4} is the natural mortality rate of calves, and PS^{VF} is the proportion of female calves sold and is defined over $[0,1]$. The total number of female calves (VFR) retained is:

$$VFR = TC(1-ER)c_{a3}(1-c_{a4})(1-PS^{VF}). \tag{34}$$

All male calves (VMS) are sold; thus, all yearlings are female. The total number of male calves sold is:

$$VMS = TC(1-ER)(1-c_{a3})(1-c_{a4}). \tag{35}$$

The total number of yearlings sold is:

$$YFS = VFR(1-c_{a5})PS^{YF}. \tag{36}$$

where c_{a5} is the proportion of yearlings that suffer from natural mortality and PS^{YF} is the proportion of female yearlings sold and is defined over $[0,1]$. The total number of yearlings retained is:

$$YFR = VFR(1-c_{a5})(1-PS^{YF}). \tag{37}$$

Consistency of the equilibrium age structure is ensured through two equations. First, all cull and dead cows are replaced each year. Thus, $TCU = TC \cdot RR$ where $RR = (1-SV)$. Second, yearlings replace cull cows in each year:

$$YFR = \sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu=2,cd,ll}. \tag{38}$$

The distribution of genetic merit across the herd in an optimal solution must be similar to those distributions observed on typical NZ dairy farms. The set of all levels of genetic merit to which a cow can belong is described $mw = [1, 2, ..., 5]$. The distribution is defined in each solution through:

$$TC \cdot me_{mw} = \sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{cu=1}^{2}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m=mw,cu,cd,ll} \quad \forall mw, \tag{39}$$

where me_{mw} is the proportion of the cows in a standard herd that possess a given level of genetic merit for milk production *mw*.

The calving distribution must also approximate those distributions observed on real farms. The fortnight corresponding to the planned start of calving (c_{a6}) is set exogenously. (However, the model is also able to optimise calving date, if this is required.) Calving is assumed to take place over the next eight weeks, spread over five fortnights. The set of all fortnights in which calving takes place is $cw = \{c_{a6}, c_{a6}+1, c_{a6}+2, c_{a6}+3, c_{a6}+4\}$. The calving distribution is enforced through:

$$TC \cdot fv_{cw} = \sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cu=1}^{2}\sum_{ll=1}^{7} C_{uf,a,m,cu,cd=cw,ll} \quad \forall cw, \tag{40}$$

where fv_{cw} is the proportion of the herd that calves in each fortnight.

The calving distribution and the time of conception should be interdependent in a static model. These are detached here to maintain model tractability. The model captures a focus on achieving high conception rates to maintain production through eq. 32. This appears as the first optimisation model to represent this feature (cf. McCall *et al.* 1999), even though a full equilibrium structure cannot be represented.

Milk production (*MP*) is defined through:

$$MP = \sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cu=1}^{2}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu,cd,ll}\, mp_{uf,a,m,cu,cd,ll}\,, \tag{41}$$

where $mp_{uf,a,m,cu,cd,ll}$ is the annual milk production (t MS^{-1}) of each cow type.

The potential intake of cows provides an upper bound to the consumption of pasture and supplement in a given period. Consumption of supplement can substitute for some of the pasture that would otherwise be ingested. This is typically described through the substitution rate, which is the kg DM reduction in potential intake for 1 kg of supplement consumed. Substitution rates vary with pasture intake, as substitution will increase as ingestion limits imposed by potential intake are approached (Stockdale, 1999). The substitution rates are computed using the regression functions of Stockdale (2000).

Two important variables impact the substitution rate. Mean intake of pasture MI_t (kg DM cow^{-1}) is computed:

$$MI_t = \frac{Q_t^H}{TC} c_{a7} c_{a8}\,, \tag{42}$$

where c_{a7} converts t DM to kg DM and c_{a8} converts the figures from per fortnight to per day.

Mean cow liveweight (ML_t) is:

$$ML_t = \frac{\sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cu=1}^{2}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu,cd,ll}\, lw_{uf,a,m,cu,cd,ll,t}}{TC}\,, \tag{43}$$

where $lw_{uf,a,m,cu,cd,ll,t}$ is cow liveweight (units of 100 kg).

The substitution rate for grass silage (SR_t^S), maize silage (SR_t^M), and PKE (SR_t^P) is:

$$SR_t^S = SR_t^M = SR_t^P = c_{a9} + c_{a10}\left[\frac{MI_t}{ML_t}\right] + c_{a11} se_t + c_{a12}\left(\frac{(F_t^S + F_t^M + F_t^P)c_{a7}}{\delta}\right) - c_{a13}^{S/M} + c_{a13}^{P}\,, \tag{44}$$

where $c_{a9} - c_{a13}$ are coefficients, δ is the days in a fortnight, se_t is an index indicating the time of year, and the sign of c_{a13} depends on supplement type. The substitution rate increases as pasture intake (the term in square brackets) grows, thereby promoting the degree that the consumption of supplement offsets potential pasture intake. For example, if computed on a daily basis, if a cow were fed 1 kg DM of maize silage daily in spring, this would offset 0.19, 0.36, 0.54, and 0.72 kg of pasture DM at intakes of 5, 10, 15, and 20 kg DM for a 480 kg cow. The substitution rate for turnips is $SR_t^T = 1$ (DairyNZ, 2010).

2.5 Integration module

Optimisation models of grazing systems involve integration of feed demand and feed supply (Figure 1). Feed demand and supply are integrated using energy and intake constraints.

Cows with different attribute combinations have different energy demands over the year. Total energy demand by the cow herd in period *t* is:

$$EC_t = \sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cu=1}^{2}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu,cd,ll}\, en_{uf,a,m,cu,cd,ll,t}\,, \quad (45)$$

where $en_{uf,a,m,cu,cd,ll,t}$ is the energy demand (MJ ME fortnight^{-1}) for a cow of a given type.

Cows with different attribute combinations also have different levels of potential intake over the year. This is measured in terms of pasture DM to ensure that intake substitution is accounted for. Total potential intake for the cow herd (t DM^{-1}) in period *t* is:

$$IC_t = \sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cu=1}^{2}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu,cd,ll}\, in_{uf,a,m,cu,cd,ll,t}\,, \quad (46)$$

where $in_{uf,a,m,cu,cd,ll,t}$ is the potential intake (t DM fortnight^{-1}) for a cow of a given type.

The energy constraint balances the demand and supply of metabolisable energy in the farming system in each period. It is defined:

$$EP_t^G + EP_t^R + EP_t^N + ES_t \geq EC_t\,. \quad (47)$$

This equation specifies that the cow herd must consume less energy than is available in each period. The shadow price of this constraint is the marginal value of energy in each period.

The intake constraint ensures that cows cannot consume impractical amounts of feed. It is:

$$IC_t \geq IN_t^{PE}\,, \quad (48)$$

where IN_t^{PE} is computed in eq. 21. Additionally, cow intake decreases at low herbage allowances. This is incorporated through inclusion of the intake model of Gregorini *et al.* (2009). The instant stocking intensity IS_t (cows ha^{-1} day^{-1}) is:

$$IS_t = \frac{TC}{RO_t}\,, \quad (49)$$

where RO_t is area grazed per day (ha day^{-1}). This is computed through:

$$RO_t = \frac{\left[\sum_{i=1}^{26}\sum_{r_i=1}^{15}\sum_{r_t=1}^{15} A_{i,t,r_i,r_t}^G\right] + A_t^R}{\delta}\,, \quad (50)$$

where δ is the days in a fortnight.

Pre-grazing herbage mass from ground level (PM_t) (t DM) is:

$$PM_t = \sum_{i=1}^{26}\sum_{r_i=1}^{15}\sum_{r_t=1}^{15} A_{i,t,r_i,r_t}^G \left(re_{r_i} + q_{i,t,r_i,r_t} + q_t^N\right) + A_{i,t,r_i,r_t}^G (re^R + q_t^R)\,, \quad (51)$$

where re_{r_i} is a residual mass (t DM ha^{-1}) and re^R is the residual for the regrassed area *AR* in period *t*. The term q_t^N in eq. 51 is the feed arising from N fertiliser application (t DM ha^{-1}) and is computed:

$$q_t^N = \frac{Q_t^N}{\left[\sum_{i=1}^{26}\sum_{r_i=1}^{15}\sum_{r_t=1}^{15} A_{i,t,r_i,r_t}^G\right]}. \tag{52}$$

Pasture utilisation (PU_t) is a concave function defined as:

$$PU_t = 1 - \exp[-K_t IS_t], \tag{53}$$

where K_t is a variable describing the shape of the function. The variable K_t is computed:

$$K_t = c_{e1}\exp[-c_{e2}DC_t], \tag{54}$$

where c_{e1} is a shape parameter and c_{e2} denotes how this variable is impacted by mean days since calving of the herd at time *t* (DC_t). Means days since calving at time *t* (days cow^{-1}) is computed:

$$DC_t = \frac{\sum_{uf=1}^{7}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cu=1}^{2}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu,cd,ll}\, dc_{cd,ll,t}}{TC}, \tag{55}$$

where $dc_{cd,ll,t}$ is the days since calving (days cow^{-1}) for each cow type in a given period.

An upper bound on the total amount of feed available to the herd (t DM) is then defined $TA_t = PM_t \cdot PU_t$. A limit on total pasture intake is then defined:

$$TA_t \geq Q_t^H. \tag{56}$$

Equations 49–56 together represent that higher stocking rates decrease intake per cow by reducing herbage allowance, but increase pasture utilisation per unit of area. These equations limit total ingestion of pasture at high stocking rates, with the associated set of post-grazing residuals across the year selected concurrently during the process of optimisation.

2.6 Pad module

Both feed and stand-off pads are represented in the model. A feed pad is a concrete area on which cows are fed for around 2–3 hours each day. A stand-off pad involves standing cows on bark chips or other soft surfaces for a proportion of the day to reduce time on pasture. Cows are not fed on a stand-off pad, in contrast to a feed pad. Keeping cows off pasture is a key strategy for reducing soil compaction and reducing nitrate leaching and nitrous oxide emissions through decreasing the amount of urine that is deposited on agricultural soils (Luo and Saggar, 2008).

The user defines if a feed pad is used through the coefficient $c_{f1} = \{0,1\}$, where 0 (1) indicates no use (use). A feed pad is automatically selected if the farm is one of production system 3–5, in line with recommended practice (C. Glassey, pers. comm.). The user defines if a stand-off pad is used through the coefficient $c_{f2} = \{0,1\}$, where 0 (1) indicates no use (use).

The coefficients $c_{f1} - c_{f2}$ greatly reduce solution time through allowing the exclusion of integer decision variables denoting the selection of either type of pad.

The proportion of the day spent on the feed pad (PR_t^{FO}) and on the stand-off pad (PR_t^{SO}) are decision variables in the optimisation. Cows obviously cannot spend more than the whole day on both pads combined. This is defined through $c_{f1}PR_t^{FO} + c_{f2}PR_t^{SO} \leq 1$. An upper bound is defined for the proportion of the day that cows may spend on the feed pad through:

$$PR_t^{FO} \leq c_{f1}c_{f3}, \tag{57}$$

where c_{f3} is the maximum proportion of the day spent on the feed pad. An upper bound is set on the proportion of the day that cows may spend on the stand-off pad through:

$$PR_t^{SO} \leq c_{f4}(c_{f1} + c_{f2}), \tag{58}$$

if $c_{f2} > 0$ and $c_{f4} = 0.5$ is the maximum proportion of the day spent on the stand-off pad if a feed pad is not present also. This equation defines that a cow cannot spend more than half of the day on a stand-off pad unless a feed pad is present, as otherwise this will lead to detrimentally low levels of feed intake.

Putting cows on pads reduces their capacity for pasture consumption. This is defined for a feed pad through:

$$IN_t^{FE}\left[1 - \exp\left(c_{f5}c_{f6}(1 - PR_t^{FO} - PR_t^{SO})\right)\right] \geq Q_t^H, \tag{59}$$

where IN_t^{FE} is computed in eq. 19, c_{f5} is a shape parameter for a nonlinear function describing the relationship between time on pasture and relative pasture intake, and c_{f6} denotes the number of hours in a day. Overall, the model provides a strong focus on cow intake. Key aspects are:

1. Total intake must be less than cow potential once substitution between feeds is taken into account. See equation 48.
2. High grazing intensity in any period reduces intake per cow through decreasing herbage allowance. See equations 49–56.
3. Cows that spend time on pads have their potential consumption of pasture reduced. See equation 59.
4. The proportion of imported feed in the total diet is constrained according to the production system simulated. See equation 22.
5. Maximum intakes of individual supplementary feeds are defined according to industry recommendations (DairyNZ 2010). See equations 23–26.

2.7 Profit module

The objective function involves maximisation of operating profit (*OP*) ($) for the farm, which is total revenue minus fixed and variable costs. Other goals are important, but maximising *OP* is the key business goal of NZ dairy farmers (Bewsell and Brown, 2011). All dollar values reported in this study are expressed in New Zealand dollars. A list of all coefficients for this module is given in Table 4.

Table 4. Standard values of coefficients in the profit module

Symbol	Coefficient	Unit	Value
c_{p1}	Milk price	$\$ t^{-1}$	$c_{p1}=5500$
c_{p2}	Price for cull cows	$\$ cow^{-1}$	$c_{p2}=700$
c_{p3}	Price for cull calves	$\$ calf^{-1}$	$c_{p3}=25$
c_{p4}	Price for cull yearlings	$\$ yearling^{-1}$	$c_{p4}=1250$
c_{p5}	Price for grass silage that is sold	$\$ t^{-1}$	$c_{p5}=300$
c_{p6}	Price for maize silage that is sold	$\$ t^{-1}$	$c_{p6}=400$
c_{p7}	Fixed costs	$\$ ha^{-1}$	$c_{p7}=900$
c_{p8}	Variable costs per cow	$\$ cow^{-1}$	$c_{p8}=500$
c_{p9}	Cost of purchasing grass silage	$\$ t^{-1}$	$c_{p9}=300$
c_{p10}	Cost of purchasing maize silage	$\$ t^{-1}$	$c_{p10}=400$
c_{p11}	Cost of purchasing PKE	$\$ t^{-1}$	$c_{p11}=350$
c_{p12}	Cost of nitrogen fertiliser	$\$ t^{-1}$	$c_{p12}=500$
c_{p13}	Cost of potassic superphosphate fertiliser	$\$ t^{-1}$	$c_{p13}=420$
c_{p14}	Application required of potassic superphosphate fertiliser per t MS	$t\ t^{-1}\ MS^{-1}$	$c_{p14}=0.8$
c_{p15}	Cost of grazing retained female calves off-farm	$\$ calf^{-1}$	$c_{p15}=6$
c_{p16}	Number of weeks that replacements are grazed off-farm	weeks	$c_{p16}=52$
c_{p17}	Transport cost for cattle grazed off-farm	$\$ head^{-1}$	$c_{p17}=9$
c_{p18}	Cost of grazing retained female yearlings off-farm	$\$ yearling^{-1}$	$c_{p18}=10$
c_{p19}	Cost of establishing and regrassing a turnip crop	$\$ ha^{-1}$	$c_{p19}=2485$
c_{p20}	Cost of establishing and regrassing a maize crop	$\$ ha^{-1}$	$c_{p20}=3300$
c_{p21}	Cost of regrassing pasture directly from pasture	$\$ ha^{-1}$	$c_{p21}=1000$
c_{p22}	Cost to establish and maintain a feed pad	$\$ ha^{-1}$	$c_{p22}=40$
c_{p23}	Cost to establish and maintain a stand-off pad	$\$ ha^{-1}$	$c_{p23}=112$

Note: All parameters are drawn from expert opinion and farm surveys.

Operating profit is defined:

$$\begin{aligned}OP = {} & c_{p1}MP + c_{p2}TCU + c_{p3}\left[VFS + VMS\right] + c_{p4}YFS + c_{p5}GSS + c_{p6}MAS - \\ & c_{p7}c_{z1} - c_{p8}TC - c_{p9}\sum_t Q_t^S - c_{p10}MAP - c_{p11}PAP - c_{p12}NF - c_{p13}c_{p14}MP - \\ & (c_{p15}c_{p16} + c_{p17})VFR - (c_{p18}c_{p16} + c_{p17})YFR - c_{p19}TU - c_{p20}MA - c_{p21}RG - \\ & c_{p22}c_{f1}TC - c_{p23}c_{f2}TC\end{aligned}, \qquad (60)$$

where c_{p1} is the milk price (\$ t^{-1} MS^{-1}), c_{p2} is the price of a cull cow (\$ hd^{-1}), c_{p3} is the price of a cull calf (\$ hd^{-1}), c_{p4} is the price of a cull yearling (\$ hd^{-1}), c_{p5} is the price of grass silage that is sold (\$ t^{-1}), c_{p6} is the price of maize silage that is sold (\$ t^{-1}), c_{p7} is the fixed cost of production (\$ ha^{-1}), c_{p8} is the variable cost of production (\$ hd^{-1}), c_{p9} is the cost of conserving grass silage (\$ t^{-1}), c_{p10} is the cost of purchased maize silage (\$ t^{-1}), c_{p11} is the cost of purchased PKE (\$ t^{-1}), c_{p12} is the cost of purchased nitrogen fertiliser (\$ t^{-1}), c_{p13} is the price of potassic superphosphate fertiliser (\$ t^{-1}), c_{p14} is the application of potassic superphosphate required per tonne of milk production (t t^{-1} MS^{-1}), c_{p15} is the cost of grazing retained female calves off-farm (\$ hd^{-1}), c_{p16} is the number of weeks that replacements are grazed off-farm, c_{p17} is the transport cost for cattle grazed off-farm, c_{p18} is the cost of grazing retained female yearlings off-farm (\$ hd^{-1}), c_{p19} is the cost of establishing and regrassing a turnip crop (\$ ha^{-1}), c_{p20} is the cost of establishing and regrassing a maize crop (\$ ha^{-1}), c_{p21} is the cost of regrassing pasture directly from pasture (\$ ha^{-1}), c_{p22} is the cost of a feed pad (\$ cow^{-1}), and c_{p23} is the cost of a stand-off pad (\$ cow^{-1}).

3. Data and Solution

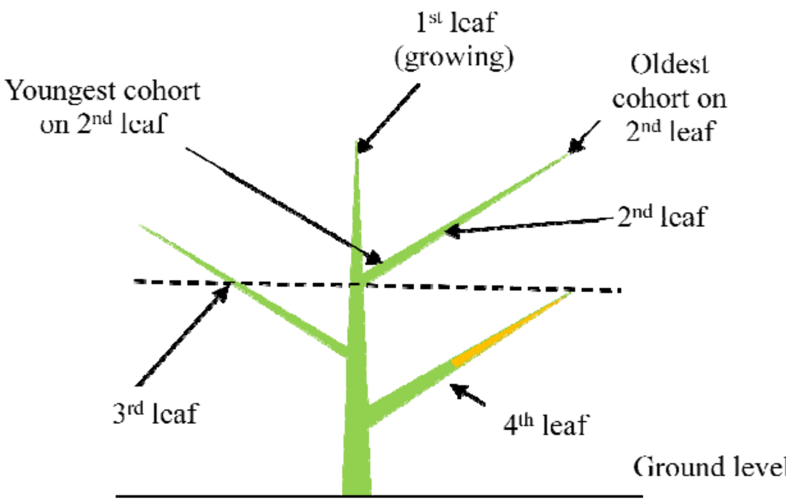
The IDEA model integrates information from a wide range of sources. Many of the coefficients are drawn from the literature and industry publications, especially DairyNZ (2010, 2011). The source of individual coefficients is listed in Tables 1–3. Objective function parameters are based on current market values in New Zealand and information from the DairyNZ Economic Survey (DairyNZ, 2011).

3.1 Estimation of pasture growth and digestibility

Information from existing pasture trials is not of sufficient quality and quantity to define pasture growth and digestibility for every time of defoliation, residual, and grazing interval combination defined in the model. A detailed simulation model of pasture dynamics that has been calibrated to appropriate experimental information and subject to extensive validation (McCall and Bishop-Hurley, 2003; Romera *et al.* 2009) is a valuable alternative, given its flexibility and rigour. This pasture model has been specifically developed and tested for perennial-ryegrass pastures in New Zealand. The model of Romera *et al.* (2009) is extended to incorporate tissue age structure to allow the estimation of pasture digestibility.

Each grass leaf is defined to consist of a series of cohorts in the age-structured version of the pasture model. A new cohort emerges each day. Each cohort possesses a thermal age, the total temperature it has experienced since emergence (measured in °C-days). A new leaf emerges every 160°C-days (phyllocron). A cohort senesces after 3 phyllocrone and remains attached to the plant for another phyllocron. Herbage intake takes please from the tip (oldest cohort) of the youngest leaf, moving down to the older leaves (always starting from the tip), one leaf after the other, until reaching the specific residual level. Figure 2 indicates how cohort age increases along a leaf because ryegrass leaves expand from growing points located at the base of the leaf. Figure 2 also shows, as an example, that all of Leaf 1, all of Leaf 2, and the oldest cohorts on Leaf 3 will be eaten if cows graze down to the particular residual shown. The digestibility of each cohort is computed as a function of thermal age using the approach of Jouven *et al.* (2006).

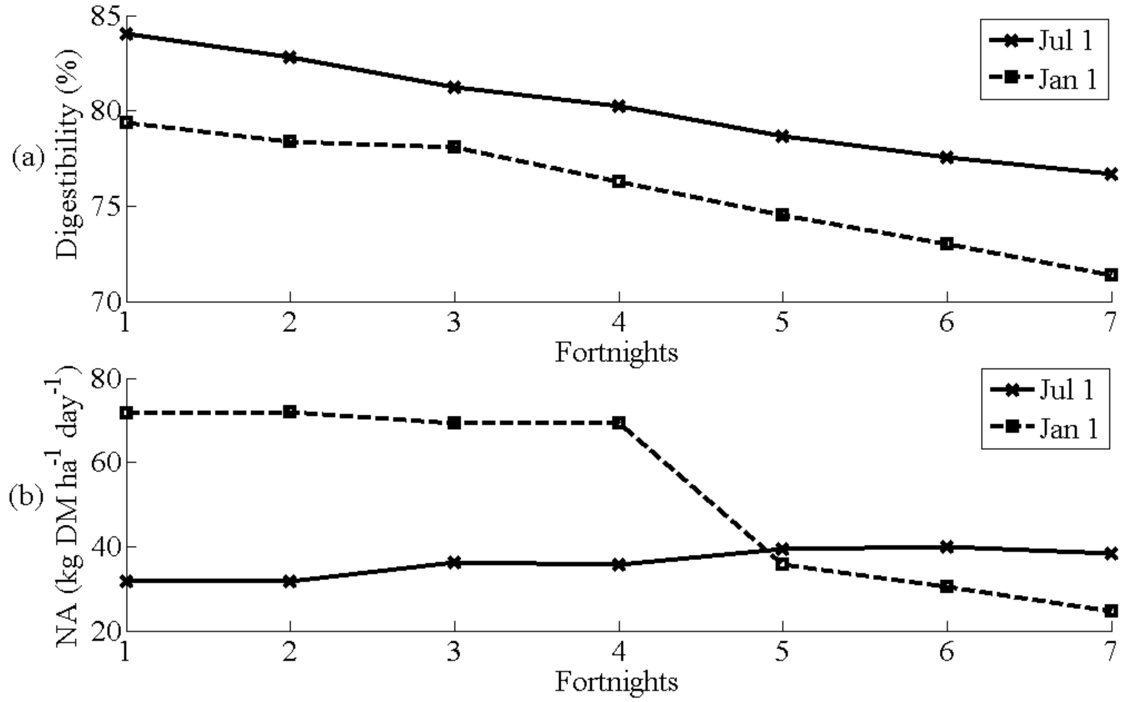
Figure 2. Diagram of a stylised perennial ryegrass plant



The pasture model generated pasture growth and pasture digestibility over time for each of the 10 post-grazing residuals ($r_i = [0.8, 1, ..., 2.6]$ t DM ha^{-1}) for each of the 26 periods across 10 individual years (2000–2009). A number of years is used to identify how age structure and pasture growth changed with climate variability. A MATLAB (Davis, 2011) program is used to process this information for entry into the optimisation model.

The pasture model allows the description of changes in the digestibility of spelled pastures across the year (Figure 3a) and seasonal decreases in pasture accumulation (Figure 3b). Digestibility decreases as the mean age of the pasture sward increases over both winter (July 1 grazing) and summer (January 1 grazing) (Fig. 3a). Digestibility is lower in summer due to the accumulation of dead material. Pasture growth is high in early summer, but growth slows in late summer due to high evapotranspiration and low rainfall. Growth in winter is reasonably consistent for this example (Fig. 3b), but is much lower than that observed over spring-early summer (e.g. Fig. 3a).

Figure 3.



Notes: (a) Pasture digestibility as a function of the fortnights since the last grazing event. (b) Net accumulation (NA), computed as growth minus senescence, as a function of the fortnights since the last grazing event.

The July 1 event involves grazing to a post-grazing residual of 1 t DM ha^{-1}. The January 1 event involves grazing to a post-grazing residual of 2 t DM ha^{-1}.

3.2 Cow model

There are a total of 17,640 attribute combinations that allow the description of a large number of different types of cow. A comprehensive optimisation model is developed to compute potential intake, energy requirements, liveweights, and empty rates for each type. The optimisation model identifies solutions that maximise milk production subject to a number of endpoint constraints. The endpoint constraints are required due to the equilibrium structure of the primary model (Section 2) and motivate the use of optimisation, rather than simulation. The cow model is based on existing simulation models, described by NRC (2001), Johnson *et al.* (2008), and Freer *et al.* (2010). Coefficients within this model have been calibrated such that the model provides a meaningful description of NZ dairy cows.

The cow is described by a given liveweight and condition score (Roche *et al.* 2009) in each period. One endpoint constraint requires that body condition is equivalent at the start and end of the year. Another requires that base liveweight increases in younger cows due to aging. Condition score decreases with milk production over the first 60 days of lactation, with the extent of loss dependent on the level of condition at calving. Condition score is never allowed to fall beneath 3.5 since lower levels are rarely observed on standard NZ dairy farms.

Potential intake for a given type of cow specifies the maximum amount of feed (in kg pasture DM) that a cow can ingest in a given period when unrestricted access is given to a feed with a digestibility of at least 80 per cent utilising the model of Freer *et al.* (2010).

Energy is allocated to gain in body condition, growth, lactation, maintenance, and pregnancy. Energy is obtained from the ingestion of feed and loss in body condition. The dynamics of body condition loss and gain are based on the comprehensive equations of NRC (2001). These deal with the storage of energy in both body fat and protein partitions. The equations that describe the energy expended for growth, lactation, maintenance, and pregnancy are based on ECOMOD (Johnson *et al.* 2008). The time after calving at which a cow first starts exhibiting oestrus is based on age and condition score at calving (Beukes *et al.* 2010).

Standard optimisation models of grazing systems typically assume that each animal can eat their potential intake in each period (e.g. McCall *et al.* 1999; Kopke *et al.* 2008). However, the energy intake of NZ dairy cows is typically reduced below potential for at least a proportion of the year, especially in the period prior to calving (Macdonald *et al.* 2008). Also, how much each cow is fed is a key decision that producers make constantly to match feed demand and supply. Accordingly, the uf subscript (Section 2.4) denotes the degree to which total energy intake over the year is constrained, relative to cows that are fully-fed. This allows for the trade-offs between production per cow and production per hectare to be explored with the model.

The cow model is solved individually for each cow type to generate the inputs required for the main model. The cow model contains 1,610 decision variables and 1,430 constraints for each attribute combination and is solved using nonlinear programming (NLP) with the CONOPT3 solver in the General Algebraic Modelling System (GAMS) Distribution 23.0 (Brooke *et al.* 2008).

3.3 Solution procedure

The IDEA model (Section 2) contains 154,300 decision variables and 8,760 constraints. It is solved using NLP with the CONOPT3 solver in GAMS. A linear version of the model is optimised before each NLP is solved to identify a good starting point for the gradient algorithm.

3.4 Model runs

Section 4.1 compares output with that from FARMAX, a relatively complex simulation model of a NZ dairy system (Bryant *et al.* 2010), and discusses base output. A representative farm is constructed based on information drawn from Waikato farms in the 2008/09 milking season from the DairyBase database (DairyNZ, 2011). The information available from survey data is not detailed enough to provide a comprehensive comparison. Thus, FARMAX is used to create a plausible representative farm to identify a set of consistent data. First, the most-profitable FARMAX scenario for a set of consistent input data, drawn from the DairyBase database and other sources, is identified through simulation. Second, equivalent input coefficients are defined in IDEA and this model is optimised. No additional constraints are defined in the model to improve its ability to match FARMAX data. The planned start of calving is assumed to be 25 July in all simulations performed in this study. A consistent calving date is used for ease of comparison between scenarios.

Section 4.2 discusses model output simulated for farms of different intensity. Section 4.3 compares model output computed for different production systems with survey data. Survey data is drawn from a sample of 71 Waikato dairy farms in the 2008/09 milking season in the DairyBase database (DairyNZ, 2011). Data for production systems 1–2 and 4–5 is pooled for parsimony.

4. Results and Discussion

4.1 Base output

IDEA provides a very good description of the farming system described by FARMAX output (Table 5). Stocking rate is identical between the two models, while milk production in IDEA is 0.6 per cent lower. This low level of disparity is important, as these variables are key drivers of profit and feed management on New Zealand dairy farms. Moreover, there is less than 1 per cent difference in lactation length, pasture eaten, and level of imported supplement. The largest difference experienced is in the level of grass silage eaten per hectare. This is larger in IDEA, but only by 20 kg. These results provide confidence in the ability of IDEA to provide insight into representative farming systems. The value of the small differences observed in Table 5 are promoted considering that output from optimisation models is seldom tested against external data for validation. (See McCall *et al.* (1999) for a rare example).

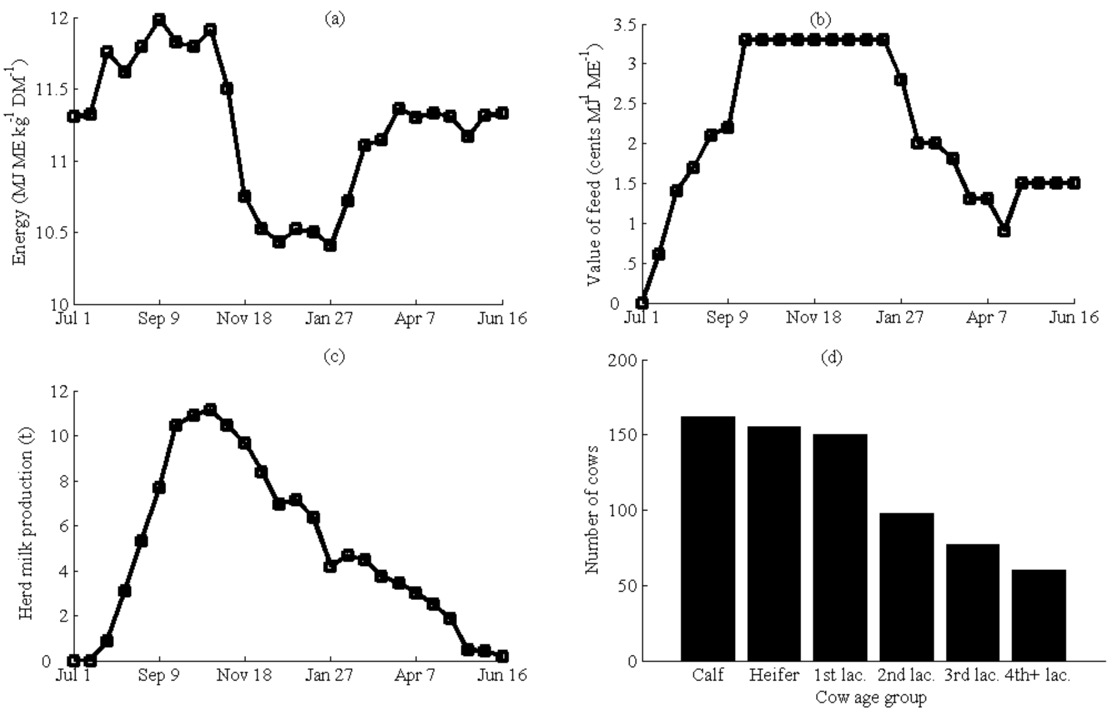
Table 5. Comparison of model output from FARMAX and IDEA

Variable	Units	FARMAX	IDEA	Diff. (%)
Imported feed	% diet	10	10	-
Farm profit	$ ha^{-1}	1201	1186	-1
Stocking rate	cows ha^{-1}	3.08	3.08	-
Milk production	kg MS cow^{-1}	333	331	-0.6
Lactation length	Days	271	272	0.37
Grazed pasture eaten	t DM ha^{-1}	12.1	12.18	0.66
Grass silage eaten	t DM ha^{-1}	0.36	0.38	5.26
Maize silage eaten	t DM ha^{-1}	0.37	0.37	0
Bought-in supp eaten	t DM ha^{-1}	1.46	1.45	-0.69
N fertiliser applied	kg N ha^{-1}	105	107	1.87
Crop area	{type, % area}	{maize, 2.7}	{maize, 2.75}	1.82
Replacement rate	%	23	21.9	-5.02

Note: This comparison is carried out for a production system 3 farm.

Figure 4 presents further output from the IDEA model obtained for the optimisation run presented in Table 5. The energy present in grazed pasture is high over late winter–spring, but declines over December–February with the accumulation of dead material in grazed herbage mass (Figure 4a). Pasture energy concentration improves over autumn-winter, relative to summer levels, but remains below the peak observed in late winter-spring. The marginal value of feed—the shadow price of eq. 47—increases steadily over July–September, before reaching a persistent peak from October to January (Figure 4b). The marginal value of feed is high over this time because individual cows reach their peak lactation at different times given a staggered calving distribution, pasture growth and quality decrease in later periods (December–January), and peak lactation for the herd is observed around the start of this persistent peak (cf. Fig. 4b and Fig. 4c). The marginal value of feed declines in autumn–winter because herd milk production declines. The age structure of the herd (Figure 4d) demonstrates that the high proportion of the milking herd are young, given the replacement rate of 21.9 per cent.

Figure 4. Model output for optimal management in the base solution



Note: (a) Pasture energy over the year. (b) The marginal value of feed over the year. (c) Herd milk production over the year. (d) Number of cows in each age group.

4.2 Value of alternative production systems

Stocking rate and milk production per cow increase with the amount of imported feed in each production system (PS) (Table 6). The increase in the amount of energy available to the herd allows lactation to be extended and promotes pasture utilisation since higher stocking rates can be supported (Table 6). The beneficial relationship between utilisation and higher stocking rates causes pasture eaten and the amount of nitrogen fertiliser applied to increase with production intensity, highlighting a complementary relationship between pasture and imported feed. The level of grass silage eaten typically decreases as the amount of imported feed increases in Table 6. This demonstrates that some substitution generally occurs between these supplements. Moreover, it is optimal on each farm to have 2.75 ha of maize silage crop and feed this as silage. However, PS5 also imports an additional 87.5 t DM of maize silage (an extra 0.7 t DM ha^{-1}), compared with PS1–PS4.

Table 6. Baseline model output for different production systems

Variable	Units	Production system				
		1	2	3 (base)	4	5
Imported feed	% diet	0	4	10	20	30
Farm profit	$ ha^{-1}	934	926	1186	1314	1093
Stocking rate	cows ha^{-1}	2.75	2.92	3.08	3.18	3.71
Milk production	kg MS cow^{-1}	294	294	331	376	378
Milk production	kg MS ha^{-1}	809	858	1019	1196	1402
Lactation length	Days	271	270	272	297	302
Grazed pasture eaten	t DM ha^{-1}	10.92	11.33	12.18	12.35	13.09
Grass silage eaten	t DM ha^{-1}	0.72	0.55	0.38	0.53	0.27
Maize silage eaten	t DM ha^{-1}	0.37	0.37	0.37	0.37	1.07
Imported supp. eaten	t DM ha^{-1}	-	0.52	1.45	3.35	5.93
Total feed eaten	t DM ha^{-1}	12.01	12.77	14.38	16.6	20.36
N fertiliser applied	kg N ha^{-1}	67	71	107	135	143
Crop area	{type, % area}	{maize, 2.75}	{maize, 2.75}	{maize, 2.75}	{maize, 2.75}	{maize, 2.75}
Replacement rate	%	21.9	21.8	21.9	22	22

Note: Output for the base farm is shaded.

Profit differs broadly across each simulated production system (Table 6). PS4 is the most profitable system, importing around 20 per cent of feed. PS1 and PS2 earn around 30 per cent less profit than PS4 because imported feed is not sufficient to support higher stocking rates and milk per cow. In contrast, PS5 is around 17 per cent less profitable than PS4, despite producing 17 per cent more milk, since high levels of costly imported feed are used. Thus, in PS5 the imported supplement is being used at a level where its marginal cost is greater than its marginal benefit. (This suboptimal result occurs given that the optimisation model is constrained to represent a given production system in each run through eq. 22.) Nonetheless, it is important to realise that these results are conditional on the exact set of agroecological and economic relationships contained in the model.

4.3 Comparison of model output for alternative production systems with survey data

IDEA exhibits a strong capacity to closely match mean information drawn from survey data (see Section 3.4) for a range of different production systems (Table 7). This is notable since each production system is quite different and survey farms are very heterogeneous, despite from being located in the same geographical region. This indicates the flexibility of the model, but also its detailed structure that helps to capture real processes not incorporated in other optimisation models of dairy systems (e.g. residual mass, rotation length) and dampen the extreme behaviour observed in linear optimisation models of agricultural systems (Howitt, 1995).

Table 7. Baseline model output for production systems 1–2, 3 and 4–5

Variable	Prod. system 1–2			Prod. system 3			Prod. system 4–5		
	Sur.	**Mod.**	**Diff.**	**Sur.**	**Mod.**	**Diff.**	**Sur.**	**Mod.**	**Diff.**
Imported feed (% diet)	2.6	2	-0.6	10.9	10	-0.9	30.3	25	-5.3
Stocking rate (cows ha^{-1})	2.6	2.84	8	3.1	3.08	-1	3.6	3.44	-5
Milk production (kg MS cow^{-1})	310	294	-5	332	331	-1	399	377	-6
Milk production (kg MS ha^{-1})	806	834	3	1029	1019	-1	1436	1297	-11
Grazed pasture eaten (t DM ha^{-1})	11.7	11.13	-5	12.7	12.18	-4	13.7	12.7	-8
Imported supp. eaten (t DM ha^{-1})	0.31	0.26	-19	1.39	1.45	4	4.15	4.64	11
Total feed eaten (t DM ha^{-1})	12.2	12.39	2	14.3	14.38	1	18.3	18.48	1
N fertiliser applied (kg N ha^{-1})	65	69	6	105	107	2	150	139	-8

Note: Columns represent means from survey data from the Waikato region (labelled ‘Sur.’), model output (labelled ‘Mod.’), and the percentage difference (labelled ‘Diff.’). The ‘Diff.’ column is shaded for each scenario.

Compared with survey data, the optimal stocking rate is 8 per cent higher in the optimisation model for PS1–2, but milk production per cow is 5 per cent lower in this scenario (Table 7). This is indicative of the inverse relationship between stocking rate and milk production per cow that exists in systems with a low level of supplementation (Macdonald *et al.* 2008). For this reason, milk production per hectare is only 3 per cent higher in the optimisation model, relative to survey data, in this scenario. Additionally, the base model (PS3) matches survey data very closely, with a maximum deviation of 4 per cent with regards to ‘grazed pasture eaten’ and ‘imported supplement eaten’. More intensive systems (PS4–5) are less well described by the optimisation model, with milk production underestimated by around 11 per cent.

Overall, only three of the differences reported in Table 7 are greater than 10 per cent. All three of these are reported for systems 1–2 and 4–5. This indicates that the model provides the best insight into a medium-intensity farm (PS3), which is expected given that it has been designed primarily to describe these systems. Two of the differences greater than 10 per cent are reported for ‘imported supplement eaten’ in Table 7. These differences arise because data for systems 1–2 and 4–5 have been combined here, but vary significantly in reality.

5. Conclusions

Pasture-based dairy farms have significant potential to contribute further to global production given increasing costs and welfare concerns associated with intensive dairy production (Dillon *et al.* 2005). However, these farms are complex systems in which producers must consider together many interdependent elements. Optimisation models are valuable tools within this context for evaluating optimal responses to new policies, prices, and technologies.
A detailed nonlinear optimisation model of a New Zealand dairy farming system is described. This framework is the first optimisation model of a grazing system to consider, both

independently and together: (1) residual mass as a decision variable of the producer, (2) pasture growth and digestibility that differ with residual pasture mass and rotation length, (3) pasture utilisation that varies by stocking rate, (4) inclusion of nonlinear functions describing substitution rates, and (5) different levels of intake regulation. Model output is shown to closely match results from a simulation model used to describe an average farm in the Waikato region of New Zealand.

The model is applied to a case study involving the relative profitability of alternative farming systems that differ according to the proportion of feed that is imported. Model output is shown to closely match survey data for alternative production systems in the Waikato region of New Zealand. A system that imports 20 per cent of feed is shown to be more profitable than systems that import more or less supplement. The increase in the amount of energy available to the herd allows lactation to be extended and promotes pasture utilisation through allowing more stock to be carried.

Overall, this paper demonstrates the increasing ability of optimisation models to incorporate important elements of complex agricultural systems that are absent from less-detailed frameworks, particularly those that are linear. Model output from the case study indicates that the inclusion of these processes allows such an optimisation model to closely resemble both simulation results and survey data.

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